

**Indian Statistical Institute, Bangalore**

B. Math. First Year, Second Semester

Probability Theory-II Back Paper Examination

Duration: 3 hours

Maximum marks: 100

Date : To be announced

1. Let  $X, Y$  be independent identically distributed random variables having uniform distribution in  $\{1, 2, 3, 4\}$ . Take  $Z = X - Y$  and  $W = X + Y$ . Find the joint distribution and marginals of  $Z, W$ . Find the conditional distribution of  $W$  given  $Z = 1$ . [15]
2. Let  $B, C$  be two independent random variables uniformly distributed in  $[-1, +1]$ . Compute the probability that the roots of the polynomial  $p(x) = x^2 + Bx + C$  are real. [15]
3. Let  $U, V$  be independent random variables each having uniform distribution in  $[0, 2]$ . Compute (i)  $P(|U - V| < 1)$ ; (ii)  $P(U + V = 3)$ ; (iii)  $P(V \geq U | U \leq 1)$ . [15]
4. Let  $X_1, X_2$  be independent  $N(0, 1)$  distributed random variables. Find the joint density of  $(X_1 + X_2, 2X_2)$ . Now find the marginal density of  $X_1 + X_2$ . [15]
5. Let  $\{X_n\}_{n \geq 1}$  be a sequence of random variables converging in distribution to a real number  $c$ . Show that  $\{X_n\}_{n \geq 1}$  converges in probability to  $c$ . [20]
6. State Central Limit Theorem (CLT) for i.i.d. random variables with finite non-zero variance. Use this theorem and statistical tables to estimate  $P(990 < S < 1020)$  where  $S$  has binomial distribution with parameters  $(1000, \frac{1}{2})$ . [20]